A Novel Optimization DMPC Algorithm with Reduced-Order Computational Complexity: Applications in IIoT-Based BMS for the Optimal Heating/Cooling of Large-scale Buildings

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***Abstract*—The present contribution reports practical deployment outcomes of an enhanced Distributed Model Predictive Control (DMPC) approach featuring diminished computational burden and superior steady-state accuracy compared to conventional Jacobi-based iterative algorithms for large-scale building thermal management. The developed methodology establishes optimal operational set points for Fan Coil Units (FCUs) alongside centralized boiler and chiller installations, thereby accomplishing thermal regulation objectives while simultaneously reducing overall building energy expenditure. Implementation is realized through resolution of constrained distributed optimization formulations executed by locally deployed intelligent thermostatic devices specifically engineered for this application. Experimental validation occurs at Sharif University of Technology's (SUT) Department of Electrical Engineering (EE) facility, encompassing eight stories across 6,700 square meters containing 111 individual spaces. Results confirm the modified algorithmic variant delivers performance metrics equivalent to (or better than) standard methodologies while achieving substantial computational efficiency gains. The investigated structure incorporates 116 networked smart thermostats implementing the proposed modified algorithm. Comprehensive performance verification through numerical simulations and field implementation demonstrates the technique's efficacy for large-scale building management systems, evidencing zero steady-state deviation and optimized energy utilization patterns.**

***Note to Practitioners*—During deployment of the DMPC technique from our prior publication within a large-scale facility (the EE Department), substantial computational burden emerged throughout individual iterations of the conventional Jacobi algorithm. Addressing this limitation, the present manuscript reformulates the traditional Jacobi iterative procedure and introduces a modified variant exhibiting reduced-order computational requirements per iteration. The methodology presented herein proves deployable across large-scale building infrastructures since its execution eliminates dependency upon high-performance and consequently costly computational servers for real-time optimization algorithm processing. The framework demonstrates elevated dependability and robustness attributable to its decentralized computational architecture, permitting implementation through readily accessible and economically viable hardware components. Superior operational characteristics of the developed technique regarding thermal management and energy expenditure optimization are validated across winter and summer operational periods via comprehensive numerical simulations alongside field deployment. The presented methodology extends beyond building automation and control applications, demonstrating relevance to additional intelligent systems incorporating convex optimization formulations within their operational frameworks.**

***Index Terms*—Standard Jacobi Iteration, Heating/Cooling System, Energy Consumption Reduction, Large-scale Systems.**

I. Introduction

A. Motivation and Backgrounds

Optimal thermal management of large-scale buildings represents a critical challenge in contemporary building automation systems, demanding sophisticated control strategies that balance occupant comfort, energy efficiency, and computational feasibility. The complexity intensifies as building scale increases, necessitating control architectures capable of real-time optimization across numerous thermal zones while operating within practical computational constraints. Existing literature addressing building thermal control predominantly employs three architectural paradigms: centralized, decentralized, and distributed approaches, each exhibiting distinct advantages and limitations. Centralized strategies, including those proposed in [1], [2], [3], and [4], offer theoretically optimal solutions by simultaneously considering all system components. However, these methodologies suffer from prohibitive computational complexity for large-scale deployments, rendering them impractical for buildings with extensive thermal zones. References [1] and [2] implement centralized MPC for building heating, demonstrating nonzero steady-state deviations, absence of meteorological forecast integration, and potentially elevated start-up energy consumption. The centralized LQR approach in [3], while incorporating weather forecasting, remains restricted to linear systems and cannot naturally accommodate operational constraints inherent to MPC formulations. Reference [4] presents centralized MPC with thermal parameter estimation filtering, yet exhibits nonzero steady-state error and excessive start-up energy demands. The work in [5] comprehensively examines MPC deployment across centralized, decentralized, and distributed architectures for building heating/cooling. This formulation employs four-term cost functions imposing substantial computational burdens requiring expensive server infrastructure for real-time decision-making in large facilities. Pre-computation

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procedures determine reference steady-state set-points u_{ref} for the cost function component $||u - u_{ref}||$. While potentially enhancing transient response under idealized conditions, conflicts emerge with the component $||y - y_{ref}||$ minimizing temperature-setpoint discrepancies, since inevitable model-plant mismatches prevent u_{ref} from precisely corresponding to y_{ref} values. This conflict generates nonzero steady-state deviations in practical deployments. Additionally, [5] addresses exclusively heating operations with potentially elevated start-up energy requirements. Decentralized control architectures, exemplified by [6], [7], and [8], distribute decision-making across individual subsystems, reducing computational complexity but sacrificing global optimality. The optimal On-Off control in [6] employs decentralized architecture potentially diminishing overall system performance by neglecting inter-subsystem thermal coupling. Furthermore, [6] lacks feedforward compensation for measurable disturbances including ambient temperature variations. Reference [7] controls individual spaces through fan velocity and FCU temperature manipulation using nonlinear dynamic models and decentralized MPC. Reference [8] presents decentralized MPC achieving zero steady-state error while incorporating meteorological forecasting. Distributed approaches, including [9], [10], [11], [12], [13], [14], and [15], theoretically balance computational tractability with near-optimal performance by coordinating multiple local optimizers. Reference [9] introduced Jacobi-based iterative techniques distributing computational loads of convex optimization problems with quadratic cost functions across distributed optimizers, demonstrating successful irrigation network applications. Building upon this foundation, [10] developed DMPC for building heating/cooling. However, [10] potentially exhibits elevated start-up energy consumption by imposing constraints exclusively on heating/cooling unit temperatures while neglecting temperature rate-of-change limitations. Furthermore, inherent trade-offs between steady-state accuracy and optimal FCU set-points generate persistent temperature-set-point discrepancies. The formulation addresses exclusively heating operations without incorporating meteorological forecasting for transient response enhancement. References [11], [12], and [13] exhibit identical limitations. Although [14] and [15] integrate weather forecast data, these approaches suffer from elevated start-up energy consumption, nonzero steady-state deviations, and heating-exclusive formulations. Industrial Internet of Things (IIoT) integration for building thermal management appears in [16], [17], and [18]. References [16] and [17] propose architectures where distributed sensors and actuators communicate with cloud servers via gateways, employing centralized MPC strategies. Consequently, these frameworks prove unsuitable for large-scale facilities while exhibiting nonzero steady-state deviations and elevated start-up energy consumption. Reference [18] examines IIoT system deployments for optimal building heating/cooling operations. Contemporary advances address scalability and performance limitations through sophisticated distributed and data-driven methodologies. [19] reviews recent advances in MPC methods for the heating/cooling systems that in the above literature review we reviewed some of them. Reference [20] developed distributed MPC-ILC for 32-zone VAV systems utilizing adjacency matrices encoding thermal zone topology, achieving 15-45% tracking performance improvements despite increased computational complexity. Reference [21] implemented distributed nonlinear MPC with ALADIN algorithms demonstrating performance comparable to centralized approaches while exhibiting significant parameter sensitivity challenges. Data-driven techniques in [22] employ two-step 2R1C models identifying building thermal characteristics from HVAC operational data, achieving thermal parameter identification errors of 21.18%, 10.86%, and 3.31%. Reference [23] developed model-based deep reinforcement learning (MBC) with ensemble neural networks and MPPI control, achieving 8.23% energy savings while reducing training data requirements by 10.52×, though validation remained simulation-based. Priority-based energy allocation through distributed MPC in [24] demonstrated computational efficiency across 3-zone and 36-zone buildings but required static priority assignments. Reference [25] integrated visual comfort constraints using linear autoregressive modeling and SVM classification at the NEST SolAce facility, reducing visual discomfort violations exceeding 90% while maintaining computational tractability. Reference [26] applied dynamic clustering algorithms to EnergyPlus simulations of 84 spaces, identifying eight distinct thermal behavior patterns enabling customized control strategies, though requiring actual building validation.

Within this context, our previous work [27] introduced enhanced DMPC based on standard Jacobi iteration [9], [10] for optimal building heating/cooling, demonstrating superiority over traditional hysteresis-based approaches regarding thermal regulation and energy consumption through extensive simulations on large-scale buildings. Satisfactory field implementation remained confined to single-room applications [27]. Upon attempting large-scale deployment at the EE Department, substantial computational burden within standard Jacobi iterations prevented achievement of simulation-quality performance, motivating the present investigation.

B. Paper Contributions

Critical examination of the comprehensive literature survey reveals existing heating and cooling system controllers predominantly exhibit either centralized architectures unsuitable for large-scale buildings, or decentralized configurations yielding suboptimal performance. Our prior publication [27] addressed this literature gap by introducing a distributed control methodology achieving zero steady-state deviation, diminished start-up energy requirements, and minimized building energy expenditure. The approach demonstrates applicability across heating and cooling seasons while incorporating meteorological forecasting. This advanced technique's satisfactory performance was validated through extensive numerical simulations and limited-scale practical deployment. However, implementation at Sharif University's EE building revealed unexpectedly substantial computational burden within standard Jacobi iteration procedures, preventing complete realization of the methodology's potential for large-scale building applications.

Addressing this limitation, the present manuscript modifies the standard Jacobi iteration procedure, introducing a reformulated variant with reduced-order computational complexity per iteration. The modified variant achieves performance characteristics closely approximating standard solutions while maintaining practical implementability. The investigated facility incorporates 116 intelligent thermostatic devices with embedded modified algorithms. Satisfactory performance for large-scale building automation exhibiting zero steady-state deviation and minimized energy consumption is validated through numerical simulations and field implementation. The methodology extends beyond Building Management Systems (BMSs) to additional intelligent systems incorporating convex optimization formulations, exemplified by [9].

C. Paper Organization

The manuscript organization follows: Section II describes the implemented IIoT system in the EE Department of Sharif University of Technology. Section III presents the standard and modified Jacobi iteration techniques. Section IV provides simulation and practical implementation results for the winter season, while Section V presents the summer season results. Finally, Section VI concludes the paper by summarizing the main contributions.

II. Implemented IIoT System in the EE Department of SUT

Contemporary Industrial Internet of Things (IIoT) applications span diverse domains including agricultural irrigation networks [9], oil and gas industry [28], and Building Management Systems (BMSs) [10] , [27]. IIoT systems deliver substantial enhancements in safety, efficiency, and productivity. Consequently, successful small-scale implementations naturally prompt large-scale expansion requests. This progression characterized the IIoT system developed by Sharif University of Technology's (SUT) EE Department for optimal heating and cooling of this large-scale facility. An initial IIoT system version developed by SUT's "IoT and Cyber-physical Lab" was deployed in a single EE building office, demonstrating promising potential for significant heating and cooling energy consumption reduction [27]. Following this achievement, Sharif University's building facilities office requested system expansion across all EE Department offices. The EE building encompasses eight stories containing 111 offices. This building's system integration represented the initial phase toward equipping all Sharif University facilities with this advanced technology, ensuring compliance with government regulations mandating minimum 20 percent building energy consumption reduction.

The EE Department's heating and cooling infrastructure comprises FCU units distributed throughout building offices alongside central boiler and chiller systems circulating hot or cold water through FCUs. Each FCU incorporates an electrical fan controlled by a thermostat. The implemented IIoT system constitutes a completely indigenous development, addressing Iran's energy imbalance crisis. The system comprises four layers: IIoT-based smart thermostats, gateways, cloud computing infrastructure, and web-based applications. Fig. 1 illustrates the smart thermostat developed for Fan Coil Unit (FCU) monitoring and control, while Fig. 2 presents the smart thermostat for split air conditioner management. Each smart thermostat incorporates sensors measuring temperature, humidity, and light intensity, alongside motion detection capabilities. Each thermostat includes an IIoT wireless modem enabling communication with floor-installed gateways. The Fig. 1 thermostat employs relays controlling FCU fans and/or electric valves; the Fig. 2 thermostat utilizes InfraRed (IR) transmitters for split air conditioner control. Each floor contains a gateway (Fig. 3) connecting floor thermostats to Internet/Intranet infrastructure and subsequently to the system's web-based application (Fig. 4). A distinctive system feature is the ECO MODE operational configuration. When thermostats operate in this mode, optimization problems are solved every five minutes. These optimization formulations encompass all building offices, regulating individual office temperatures around desired set-points while simultaneously optimizing central boiler and chiller temperatures [27]. This optimization generates optimal set-points for FCU units and building central heating/cooling systems.

Conventional centralized optimization techniques, including active-set and interior-point methods [29], [30], could theoretically solve this optimization problem. However, prohibitive computational complexity prevented timely problem resolution [30] for the EE Department. This difficulty was anticipated given the EE Department's large-scale nature (numerous offices) and the frequently-solved constrained optimization formulation. Addressing this computational scalability challenge, we developed Jacobi-based iterative optimization techniques in [9], demonstrating successful application to optimal heating/cooling of large-scale buildings in [10] and [27] through extensive numerical simulations. This technique theoretically enables constrained optimization problem resolution by the distributed smart thermostats illustrated in Fig. 1 and 2. However, standard Jacobi-based iterative optimization applied to these smart thermostats produced excessive computation times when solving reduced-order optimization problems in individual iterations. Extended computation times result from limited smart thermostat computational capacity and EE building scale. Consequently, contrary to expectations, the methodologies proposed in [10] and [27] prove impractical for large-scale buildings. Addressing this limitation, this manuscript presents a modified standard Jacobi-based iterative optimization variant, demonstrating satisfactory implementation in large-scale facilities such as the EE Department of SUT.

III. The standard and modified version of the Jacobi iterative optimization algorithm

A. The Standard Jacobi Iterative Optimization Algorithm

The distributed optimization methodology proposed in [9] and [10] solves the following convex optimization problem, where $J(.)$ represents a convex function of decision variables: u_1, u_2, ..., u_n, where n denotes the number of smart thermostats in the building, u_i represents the airflow temperature

Fig. 1. The IIoT-based thermostat for controlling FCUs

Fig. 2. The IIoT-based thermostat for controlling split air conditioners

Fig. 3. The gateway used in each floor of the building

Fig. 4. The web-based application

of the FCU controlled by the i-th smart thermostat, and $\{\mathcal{U}_i\}$ are convex sets describing the constraints on u_is.

$$\min_{u_i \in \mathcal{U}_i} J(u_1, u_2, ..., u_n) \tag{1}$$

To solve this constrained convex optimization problem, an iterative technique was proposed in [9] and [10]. At the t-th iteration, each thermostat (e.g., the j-th thermostat) simultaneously solves the following reduced-order optimization problem by focusing exclusively on its decision variable while fixing other thermostats' decision variables communicated in the previous iteration:

$$h_j^* = \arg\min_{h \in \mathcal{U}_j} J(u_1^{t-1}, ..., u_{j-1}^{t-1}, h_j, ..., u_n^{t-1}), \tag{2}$$

Subsequently, $u_j^t = \lambda_j h_j^* + (1 - \lambda_j) u_j^{t-1}$, where λ_j is fixed a priori such that $\sum_{j=1}^{n} \lambda_j = 1$.

For sufficiently small $\epsilon > 0$, suppose t_ϵ represents the smallest integer satisfying:

$$|J(u_1^{t_\epsilon - 1}, ..., u_n^{t_\epsilon - 1}) - J(u_1^{t_\epsilon}, ..., u_n^{t_\epsilon})| \leq \epsilon.$$

Reference [9] demonstrates that $u_1^{t_\epsilon}$, ..., $u_n^{t_\epsilon}$ constitute an acceptable approximation of the unique optimal solution to optimization problem (1). Specifically, $(u_1^{t_\epsilon}, ..., u_n^{t_\epsilon})$ provides an acceptable approximation when t_ϵ is sufficiently large (e.g., $t_\epsilon \geq 50$). Reference [27] presented a DMPC method utilizing this standard Jacobi iterative optimization technique within each receding horizon to determine optimal solutions. For very small-scale applications (i.e., single office), [27] demonstrated this distributed optimization technique achieved temperature regulation with zero steady-state error and reduced heating/cooling energy consumption.

The receding horizon nature necessitates frequent solution updates (e.g., every five minutes) with limited allowable computation time (e.g., one minute). For large-scale buildings, such as the EE Department with 116 smart thermostats (i.e., n = 116), the time required for solving optimization problem (2) becomes excessive. Although (2) constitutes a single-variable optimization problem, floating-point computations involving 115 decision variables communicated from other thermostats prove time-consuming due to smart thermostat computational limitations. Consequently, completed iterations within one minute remain minimal (e.g., five iterations). The distributed optimization solution from [10] and [27] obtained after one minute (the guaranteed time slot) remains far from optimal. Sections IV and V demonstrate this premature solution yields poor temperature regulation performance.

B. The Modified Version

Addressing this limitation, we propose a modified standard Jacobi iterative optimization technique. For each office (e.g., the j-th office), we determine neighboring offices exhibiting strong thermal interaction by examining the building's thermal model. Subsequently, in optimization problem (2), we consider the modified cost function $J(.)$ including exclusively decision variables $(u_i s)$ of offices with strong thermal coupling to the j-th office. Consequently, optimization problem (2)'s computational complexity reduces significantly, since each office demonstrates strong thermal interaction with only a few offices. This method additionally achieves substantial communication load reduction compared to standard Jacobi iterative optimization. Using this modified version, completed iterations within the guaranteed time slot increase significantly due to substantial computational complexity reduction in optimization problem (2) solved by distributed smart thermostats per iteration. Simulation and practical implementation results in subsequent sections demonstrate the proposed modified version achieves performance closely approximating the ideal case proposed in [10] and [27] for large-scale buildings.

Implementing this method requires extracting building thermal dynamics using first-principle thermal relations combined with data-driven techniques. Each office's thermal behavior is described by an RC model [10], where R_a represents thermal resistance between office and outside, R_n represents thermal resistance between office and neighboring offices, and C represents the office's thermal capacity for temperature preservation. The office receives FCU heating/cooling airflow represented by a current source with current value $c_p \rho \dot{V}(u - T)$, where c_p is air specific heat capacity, ρ is air density, $\dot{V}$ is volumetric flow rate, u is FCU airflow temperature, and T is office temperature. Using this descriptive model, each office's thermal behavior is:

$$C\dot{T} + \frac{T - T_a}{R_a} + \frac{T - T_n}{R_n} = c_p \rho \dot{V}(u - T), \tag{3}$$

where T_a is outside temperature and T_n is surrounding neighboring offices' temperature. In (3), C depends on office volume, R_a depends on window thickness and area, and R_n depends on wall thickness, materials, and total area. Additionally, $\dot{V}$ is available from FCU data sheet. These parameters enable determining R_a, R_n, and C for each office. However, for the EE Department, determining C, R_a and R_n using these parameters yielded poor models. Therefore, we developed a data-driven technique determining C, R_a and R_n verified to produce accurate models. These results appear in another paper.

Briefly, our data-driven technique operates as follows: To determine R_a, R_n, C using data-driven approaches, we conduct three experiments employing history matching techniques to determine R_a, R_n and C such that the curves from experimental results fit curves from models based on R_a, R_n and C. Specifically, we conduct three experiments per office: Action, no action, and ECO MODE experiments. For action experiments, FCU operates for specific periods while recording office temperature, outside air temperature, building average temperature, central boiler or chiller temperature, and FCU airflow temperature. Subsequently, FCU deactivates while recording these parameters, except FCU airflow temperature, which equals office temperature. Following this, using history matching techniques, we determine R_a, R_n and C for each office so experimental temperature curves fit curves from estimated R_a, R_n and C. Subsequently, denoting office temperature as the state variable, FCU airflow temperature u as control signal, and office temperature as observation signal, we obtain the state-space thermal building model. Specifically, having C, R_a, and R_n for each office, let $u = [u_1 \; u_2 \; ... \; u_{116}]^T$, $y = [T_1 \; T_2 \; ... T_{116}]^T$ and $x = y$. We embed the obtained model into office smart thermostats via the web-based application. Subsequently, we simulate ECO MODE response using the obtained model and compare with practical response (ECO MODE experiment) for verification.

To upgrade this building's heating/cooling system, existing traditional hysteresis-based thermostats were replaced by 116 smart thermostats shown in Fig. 1; each floor received the gateway shown in Fig. 3. Additionally, building central chiller and boiler were equipped with IIoT-based smart devices specially developed for monitoring and tuning central chiller and boiler temperature based on recommendations from the system's web-based application dictating DMPC method results from [27], solved using the proposed modified Jacobi iterative optimization algorithm. Having C, R_a, and R_n for each office, let $u = [u_1 \; u_2 \; ... \; u_{116}]^T$, $y = [T_1 \; T_2 \; ... T_{116}]^T$ and $x = y$. The discrete-time building thermal model is:

$$x[k+1] = Ax[k] + Bu[k] + Ed[k], \tag{4}$$

where A is a 116×116 known matrix, B is a 116×116 known matrix, E is a 116×3 known matrix, $k = 1, 2, 3, ...$ minutes, and $d = [T_a \; T_o \; T_f]^T$, where T_o is uncontrolled area temperature (stairs, etc.) and T_f is ground temperature. Having this model, to determine offices with strong thermal interaction with the j-th office, we expand the thermal dynamics (e.g., eq. 4):

$$x[k] = A^k x[0] + \sum_{i=1}^{k-i-1} A^{k-i-1} Bu[i]. \tag{5}$$

The term $\sum_{i=1}^{k-i-1} A^{k-i-1} B$ represents thermal interactions between offices. Since A is stable (i.e., all eigenvalues inside the unit circle), the term $\mu_\infty = \lim_{k \to \infty} \sum_{i=1}^{k-i-1} A^{k-i-1} B$ exists and is k-independent. Let μ_a be a matrix where $\mu_a(j,i) = |\mu_\infty(j,i)|$. We define the Interaction Strength (IS) matrix with elements $IS(j,i)$:

$$IS(j,i) = \frac{\mu_a(j,i)}{max_i(\mu_a(j,i))}. \tag{6}$$

For the EE Department, IS is a 116×116 known matrix with diagonal elements equaling 1 and the off-diagonal elements less than 1.

To identify offices with strong thermal interactions with the j-th office, we examine the j-th row of the interaction strength matrix IS. Element $IS(j,j)$ representing the j-th office's self-interaction equals 1; offices with strong thermal interaction have corresponding elements $IS(j,i)$ larger than other row elements. To identify these offices precisely, we set threshold η yielding at most 4 offices with strong interaction, selecting offices with corresponding element $IS(j,i) > \eta$ as offices with strong thermal coupling to the j-th office. For the EE Department, most offices have strong thermal interaction limited to two or three offices; office No. 405, office No. 406, and office No. 409 have the largest neighboring office count with strong interactiononly four offices. Therefore, the proposed modified standard Jacobi iterative optimization method achieves significant computation load reduction associated with optimization problem (2) by eliminating unnecessary floating-point computation associated with decision variables of offices with weak or no interaction with the j-th office.

Algorithm1 presents the proposed modified Jacobi-based iterative DMPC algorithm.

Algorithm 1 Modified Jacobi-based Iterative Algorithm

For subsystem i:
Step0: Set $k = 0$, and set the initial assumed states of the coupled neighboring subsystems.
Step1: Solve the optimization problem at time step k with the initial state $x_i(0)$ and the assumed states of its coupled neighboring subsystems, to obtain optimal control sequence: $u_i^*[k]$, and optimal state sequence: $x_i^*[k]$.
Step2: Send the optimal state sequence to the coupled neighbors, and receive theirs.
Step3: Check the stop condition: $|J[k-1] - J[k]| \leq \epsilon$.
if stopping condition = False **then**
    Step4 Go back to Step1.
else
    Step5: Apply the first element of optimal control sequence: $u_i^*[k]$ to the system and set $x_i[0] = x_i^*[k]$.
    Step6 Move horizon: $k = k+1$, and go to Step1.

Our published paper [27] compared the DMPC method using standard Jacobi iterative algorithm with the benchmark hysteresis-based method for large-scale building temperature regulation without considering computational complexity and communication overhead. Reference [27] demonstrated this DMPC method's promising performance over the benchmark approach. Our paper [10] compared this DMPC method with commonly used centralized MPC for large-scale building temperature regulation, demonstrating DMPC method superiority over centralized MPC. However, implementing this DMPC method on the case study building, a large-scale facility, revealed computational and communication complexity challenges. Addressing this limitation, this paper proposes a modified Jacobi iterative method. Simulations in subsequent sections (Section IV and Section V) demonstrate that DMPC method performance equipped with the modified Jacobi iterative algorithm approximates DMPC method performance in [10] and [27], indicating the proposed method's superiority over benchmark methods for large-scale building temperature regulation with minimum energy consumption.

Table I compares standard and modified Jacobi iterative optimization algorithms. In standard Jacobi, each thermostat communicates with $(n-1)$ other thermostats and includes their decision variables in its m floating-point computation instructions. Therefore, total work per round = 116 thermostats × 115 considerations = 13,340 calculations per round in standard Jacobi algorithm. In the modified version, each thermostat communicates with only k neighboring offices with strong interactions. On average, $k \approx 3$. Therefore, total work per round ≈ 116 thermostats × 3 considerations ≈ 348 calculations per round in modified Jacobi algorithm. Hence, communication, computation, and memory usage improvement factor in the modified version is $13,340 \div 348 \approx 38$ in this case study.

C. The Convergence And Stability Analysis

The following lemmas establish feasibility and convergence of the proposed modified version.

TABLE I
STANDARD AND MODIFIED JACOBI COMPARISON

Metric	**Standard Jacobi**	**Modified Jacobi**	**Improvement Factor**
Communication	$O(n^2)$	$O(nk)$	$n/k \approx 38$x
Computation	$O(n^2 m)$	$O(nkm)$	$n/k \approx 38$x
Memory usage	$O(n)$	$O(k)$	$n/k \approx 38$x
Total Time	T_1	$T_2 \approx T_1/38$	$\approx 38\times$ (empirical)

Lemma1 (Feasibility). Given a feasible initial value and convex constraint sets, the iterates satisfy $(u_1^t, ..., u_n^t) \in \mathcal{U}_1 \times ... \times \mathcal{U}_n$ for all $t \geq 1$.

Proof: Following the assumptions, the initial value is selected from the feasible sets, so $(u_1^0, ..., u_n^0)$ is feasible. Since constraint sets $\mathcal{U}_1, ..., \mathcal{U}_n$ are convex, the convex combination $u_j^t = \lambda_j h_j^\star + (1-\lambda_j) u_j^{t-1}$ with $t = 0$ yields $(u_1^1, ..., u_n^1)$ is feasible, where $h_j^\star$ is the solution of (2) by considering the modifications of the modified version. For $t > 1$, feasibility follows by induction. □

Lemma2 (Convergence). Suppose $J(.) \geq 0$ is a smooth convex function of its variables. Then, for each iteration $t > 0$ the cost function $J(u_1^t, ..., u_n^t)$ is non-increasing and therefore converges as $t \to \infty$ following the Monotone Convergence Theorem.

Proof: For every iteration ($t > 0$), the cost function satisfies:
$J(u_1^t, ..., u_n^t) = J\Big(\lambda_1 h_1^\star + (\lambda_2 + ... + \lambda_n) u_1^{t-1}, ..., \lambda_n h_n^\star + (\lambda_1 + ... + \lambda_{n-1}) u_n^{t-1}\Big) = J\Big(\lambda_1 (h_1^\star, u_2^{t-1}, ..., u_n^{t-1}) + \lambda_2 (u_1^{t-1}, h_2^\star, ..., u_n^{t-1}) + ... + \lambda_n (u_1^{t-1}, u_2^{t-1}, ..., h_n^\star)\Big) \leq \lambda_1 J(h_1^\star, u_2^{t-1}, ..., u_n^{t-1}) + \lambda_2 J(u_1^{t-1}, h_2^\star, ..., u_n^{t-1}) + ... + \lambda_n J(u_1^{t-1}, u_2^{t-1}, ..., h_n^\star) = \lambda_1 J(h_1^* + \delta_1, u_2^{t-1}, ..., u_n^{t-1}) + \lambda_2 J(u_1^{t-1}, h_2^* + \delta_2, ..., u_n^{t-1}) + ... + \lambda_n J(u_1^{t-1}, u_2^t, ..., h_n^* + \delta_n) \approx \lambda_1 J(h_1^*, u_2^{t-1}, ..., u_n^{t-1}) + \lambda_2 J(u_1^{t-1}, h_2^*, ..., u_n^{t-1}) + ... + \lambda_n J(u_1^{t-1}, u_2^{t-1}, ..., h_n^*) \leq \lambda_1 J(u_1^{t-1}, u_2^{t-1}, ..., u_n^{t-1}) + \lambda_2 J(u_1^{t-1}, u_2^{t-1}, ..., u_n^{t-1}) + ... + \lambda_n J(u_1^{t-1}, u_2^{t-1}, ..., u_n^{t-1}) = (\lambda_1 + \lambda_2 + ... + \lambda_n) J(u_1^{t-1}, u_2^{t-1}, ..., u_n^{t-1}) = J(u_1^{t-1}, ..., u_n^{t-1})$, where the first inequality follows from the cost function $J(.)$ convexity, δ_j is the small difference between the solutions $h_j^*$and $h_j^\star$ and the fourth equity follows from the smoothness of the cost function and the fact that δ_j is very small following the procedure that we calculate h_j^*. Subsequently, since h_j^* is optimal, $J(u_1^{t-1}, ..., h_j^*, ..., u_n^{t-1}) \leq J(u_1^{t-1}, ..., u_j^{t-1}, ..., u_n^{t-1})$. The final equality follows from the assumption $\lambda_1 + \lambda_2 + ... \lambda_n = 1$. Since (as shown above) $J(u_1^t, ..., u_n^t)$ is a non-increasing function of t and $J \geq 0$ has a lower bound (i.e., 0), the Monotone Convergence Theorem establishes convergence as $t \to \infty$ [31]. □

Exponential stability follows from similar arguments used in [32] under the assumptions of the existence of bounds for

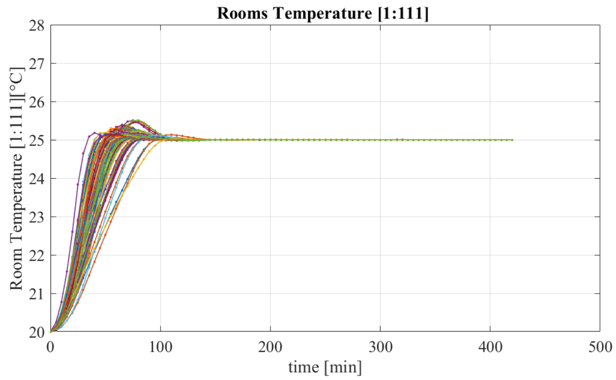

Fig. 5. The temperature regulation performance of the standard Jacobi-based iterative optimization algorithm, as proposed in [10], [27] for winter season, without considering smart thermostats computational limitation

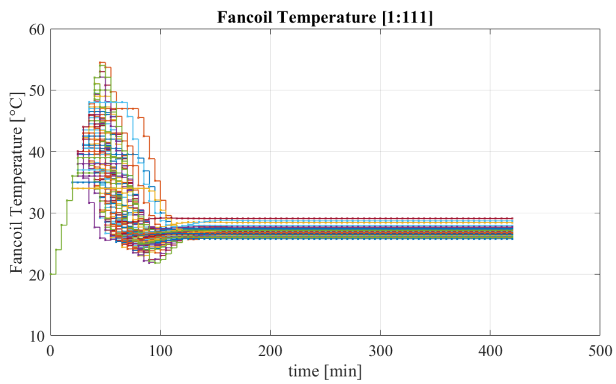

Fig. 6. FCUs temperature set by the standard Jacobi-based iterative optimization algorithm, as proposed in [10], [27] for winter season, without considering smart thermostats computational limitation

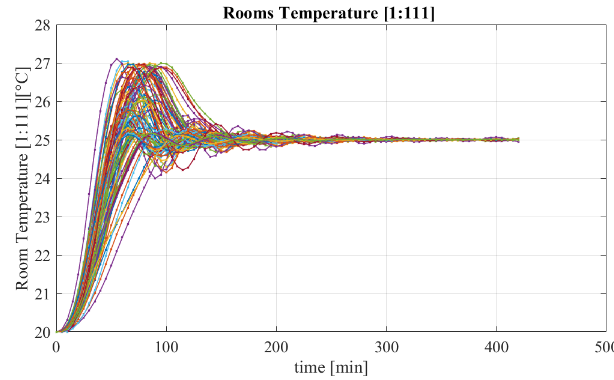

Fig. 7. The temperature regulation performance of the standard Jacobi-based iterative optimization algorithm, as proposed in [10], [27] for winter season, when we consider smart thermostats computational limitation

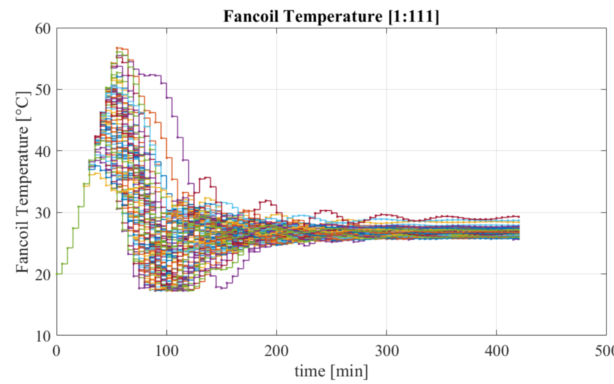

Fig. 8. FCUs temperature set by the standard Jacobi-based iterative optimization algorithm, as proposed in [10], [27] for winter season, when we consider smart thermostats computational limitation

control actions u_js and the existence of the steady - state solution for the underlying system, where for the case study system these assumptions are satisfied as the underlying dynamic system is a stable system and we also consider allowable bounds for u_j as constraints.

IV. Simulation and Practical Implementation Results for Winter Season

The proposed IIoT-based BMS system was successfully implemented in the EE Department, an 8-story building with 6,700 square-meter area and 111 offices equipped with 116 smart thermostats shown in Fig. 1. During performance evaluation, outside temperature was 0°C, office initial temperature was approximately 20°C and desired temperature was 25°C. Testing occurred from 00:00 to 07:00 during winter season under ECO MODE operation. Fig. 5 simulates temperature regulation performance when the original standard Jacobi-based iterative optimization algorithm from [10], [27] is used without considering smart thermostats' computational limitations causing premature optimization solutions. Specifically, simulations employed $t_\epsilon = 50$ instead of $t_\epsilon = 5$.

Fig. 6 illustrates building FCU temperatures tuned by smart thermostats for this case. Figs. 7 and 8 simulate the temperature regulation performance when standard Jacobi-based iterative optimization algorithm is used with smart thermostat computational limitations considered in simulations; consequently, t_ϵ is limited to five. Figs. 9 and 10 shows performance when the proposed modified optimization technique is employed. Due to significant computational complexity reduction, t_ϵ can be set to 50. For office No. 623 and office No. 624, Figs. 11, 12, 13, and 14 compare simulation results with practical implementation results. These figures demonstrate that for the proposed modified optimization technique, practical implementation and simulation results are consistent for room temperature regulation. However, regarding FCU temperatures (control action), simulation results slightly outperform practical implementation results, as expected. This holds for other offices; however, due to space limitations, practical implementation results for the remaining 109 offices are not presented.

Figs. 5 and 6 illustrate IIoT-based BMS system performance equipped with the standard Jacobi iterative technique without considering smart thermostats computational limitations. Therefore, we designate this the ideal case. From Figs. 5, 6, 9 and 10, IIoT-based BMS system performance equipped with the proposed modified optimization technique approximates the ideal case. However, due to high computational complexity associated with optimization problem (2), when IIoT-based

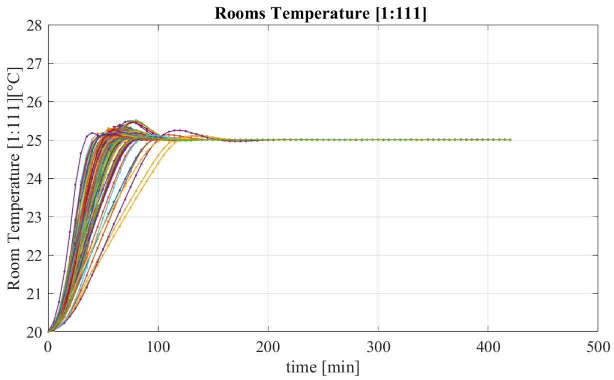

Fig. 9. The temperature regulation performance of the proposed modified version optimization algorithm for winter season

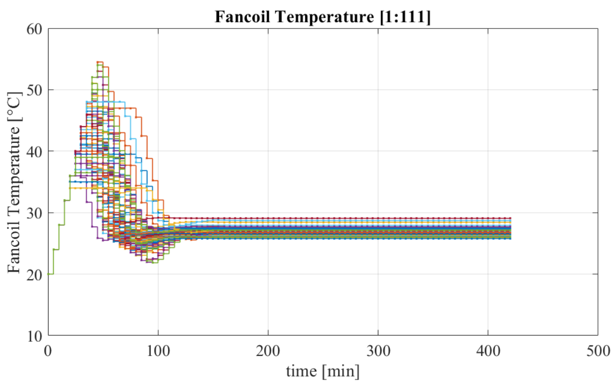

Fig. 10. FCUs temperature set by the proposed modified version optimization algorithm for winter season

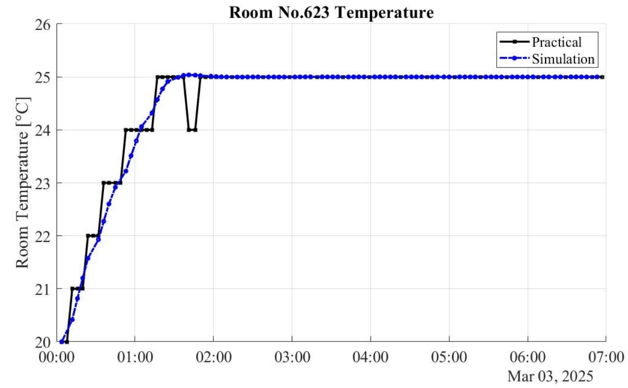

Fig. 11. The temperature regulation performance of the proposed modified version optimization algorithm for the office No. 623 for winter season

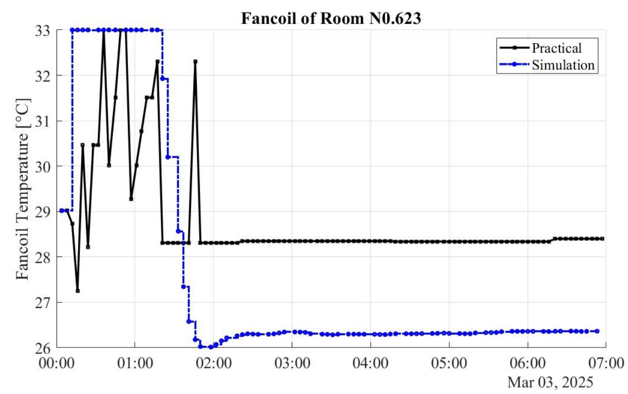

Fig. 12. FCUs temperature set by the modified version optimization algorithm for the office No. 623 for winter season

BMS system equipped with standard Jacobi optimization technique is implemented in large-scale buildings, such as the EE Department with 116 smart thermostats, standard Jacobi optimization technique iterations must terminate quickly to meet the short guaranteed time slot. Consequently, temperature regulation quality is poor, as shown in Figs. 7 and 8. Fig. 9 demonstrates all EE building office temperatures reach desired 25°C after 140 minutes. Subsequently, Fig. 10 shows maximum FCU temperatures are 28°C, meaning central boiler temperature can reduce from 65°C to 40°C while each office maintains desired temperature. Note that central boiler temperature must slightly exceed 28°C to compensate for thermal losses in transmission pipelines. Reducing boiler temperature by 25°C, achieved by the proposed IIoT-based BMS system equipped with the proposed optimization technique in ECO MODE, results in approximately 50% gas reduction for building heating services.

V. Simulation and Practical Implementation Results for Summer Season

During summer season performance evaluation, outside temperature was 33°C, office initial temperatures were approximately 29°C, and desired temperature was 25°C. Testing occurred from 05:00 to 10:00 under ECO MODE operation. Fig. 15 simulates temperature regulation performance of the original standard Jacobi-based iterative optimization algorithm for the ideal case (i.e., without considering smart thermostat computational limitations). Specifically, simulations employed $t_\epsilon = 50$ instead of $t_\epsilon = 5$. Fig. 16 illustrates building FCU temperatures tuned by smart thermostats for this case. Figs. 17 and 18 simulate temperature regulation performance when standard Jacobi-based iterative optimization technique is used with smart thermostat computational limitations considered; consequently t_ϵ is limited to five. Figs. 19 and 20 show performance when the proposed modified optimization technique is employed. Due to significant computational complexity reduction, t_ϵ can be set to 50. For office No. 623 and office No. 624, Figs. 21, 22, 23, and 24 compare simulation results with practical implementation results. These figures demonstrate that for the proposed modified optimization technique, practical implementation and simulation results are consistent. This holds for other offices; however, due to space limitations, practical implementation results for the remaining 109 offices are not presented.

Figs. 15 and 16 illustrate IIoT-based BMS system performance for the ideal case. From Figs. 15, 16, 19 and 20, IIoT-based BMS system performance equipped with the proposed modified optimization technique approximates the ideal case. However, due to high computational complexity, IIoT-based BMS system equipped with standard Jacobi optimization technique is not implementable in large-scale buildings, such as

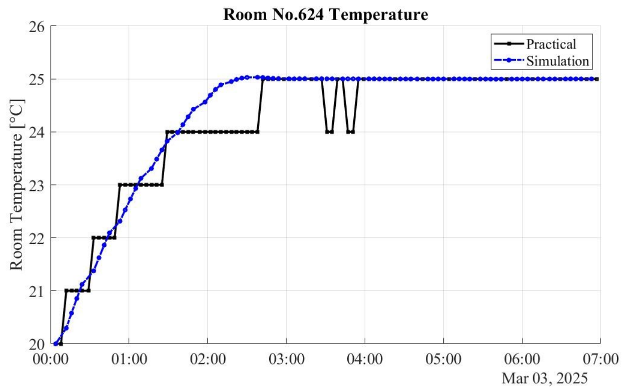

Fig. 13. The temperature regulation performance of the proposed modified version optimization algorithm for the office No. 624 for winter season

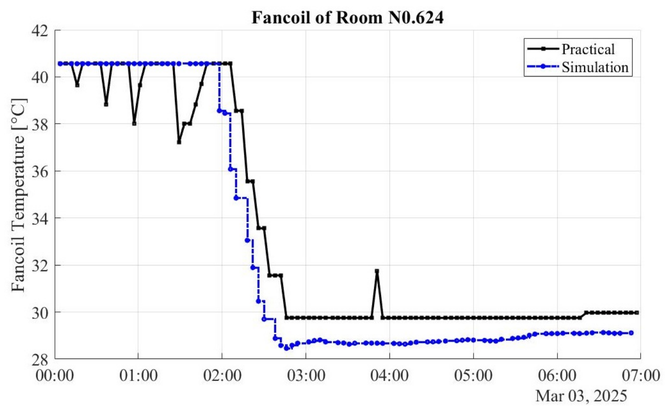

Fig. 14. FCUs temperature set by the modified version optimization algorithm for the office No. 624 for winter season

the EE building with 116 smart thermostats, resulting in poor performance as shown in Figs. 17 and 18. Fig. 19 demonstrates all EE building office temperatures reach desired 25°C after 140 minutes. Subsequently, Fig. 20 shows maximum FCU temperatures are set to 23°C, meaning central chiller temperature can increase from 9°C to 16°C while each office maintains desired temperature. Increasing central chiller temperature by 7°C, achieved by the proposed IIoT-based BMS system equipped with the proposed optimization technique in ECO MODE, results in approximately 50% electricity reduction for building cooling.

VI. CONCLUSION

Our previously published paper [27] presented an advanced DMPC method based on standard Jacobi iteration for optimal heating/cooling of large-scale buildings. Extensive computer simulations demonstrated this method's superiority over traditional hysteresis-based methods regarding energy consumption and temperature regulation performance around desired set-points. We additionally demonstrated superiority over other advanced methods concerning steady-state error, start-up energy consumption, and applicability for both summer and winter seasons. Unexpectedly, the method in [27] proved inapplicable to large-scale buildings in practice. Therefore, this paper modified the standard Jacobi iterative technique, presenting a

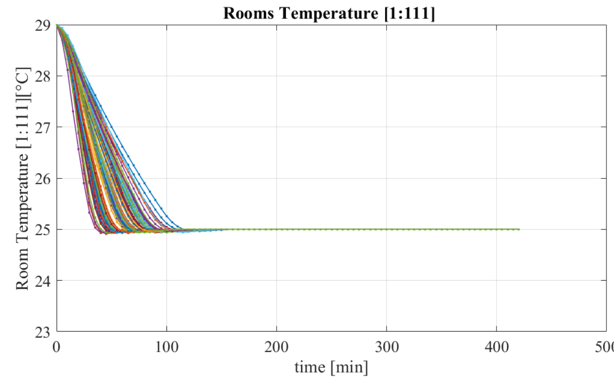

Fig. 15. The temperature regulation performance of the standard Jacobi-based iterative optimization algorithm, as proposed in [10], [27] for summer season, without considering smart thermostats computational limitation

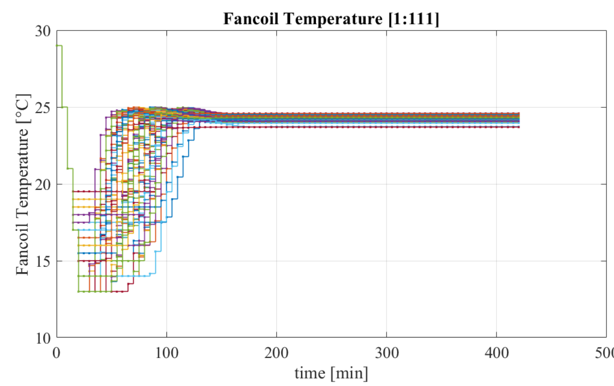

Fig. 16. FCUs temperature set by the standard Jacobi-based iterative optimization algorithm, as proposed in [10] , [27] for summer season, without considering smart thermostats computational limitation

modified variant with reduced computational complexity per iteration. Subsequently, we demonstrated satisfactory DMPC method performance equipped with this modified technique through extensive computer simulations and practical implementation by deploying this method to SUT's EE Department. The proposed method applies not only to building automation systems but also to other smart systems involving convex optimization problems in their formulations.

Real-world implementation and practical deployment revealed important lessons: implementing, testing, and debugging the system in relatively small stages (e.g., individual building floors), then integrating and deploying the complete system, proved essential. Additionally, providing methods for remotely upgrading embedded firmware of distributed smart thermostats proved helpful and improved deployment time. Future extension opportunities include integrating existing heating and cooling systems with renewable energy sources.

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The authors used Claude (Anthropic) to improve the grammar, style, and readability of this manuscript. All research design, methodology, experimental work, data collection, analysis, results, interpretations, and intellectual contributions are solely the original work of the authors.

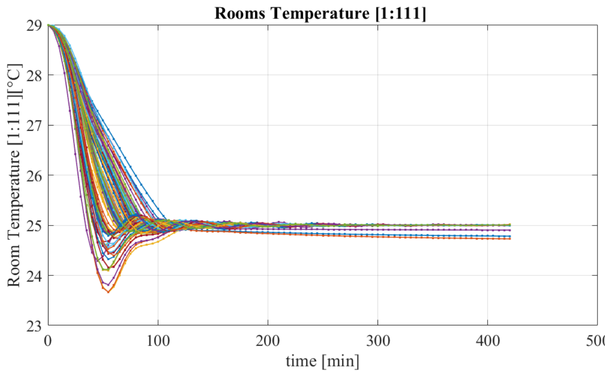

Fig. 17. The temperature regulation performance of the standard Jacobi based iterative optimization algorithm, as proposed in [10], [27] for summer season, when we consider smart thermostats computational limitation

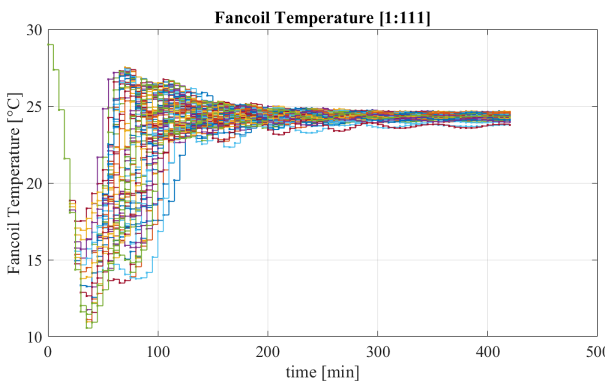

Fig. 18. FCUs temperature set by the standard Jacobi-based iterative optimization algorithm, as proposed in [10], [27] for summer season, when we consider smart thermostats computational limitation

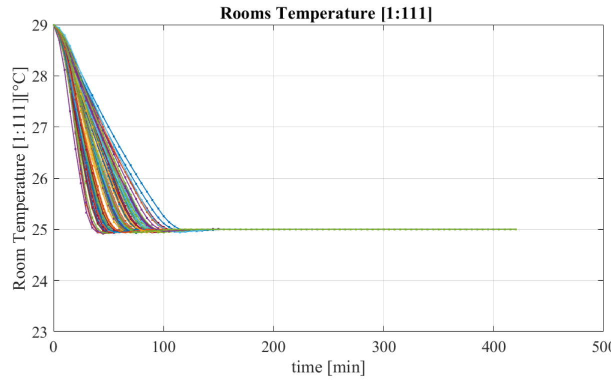

Fig. 19. The temperature regulation performance of the proposed modified version optimization algorithm for summer season

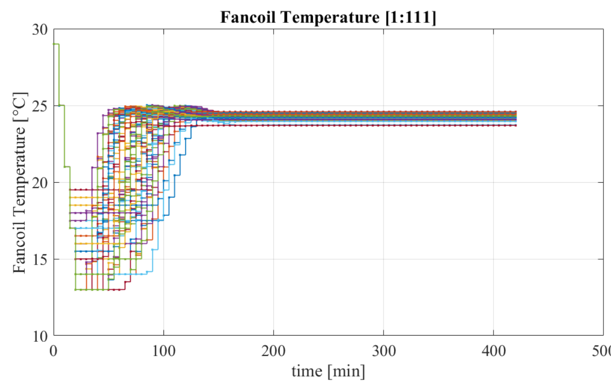

Fig. 20. FCUs temperature set by the proposed modified version optimization algorithm for summer season

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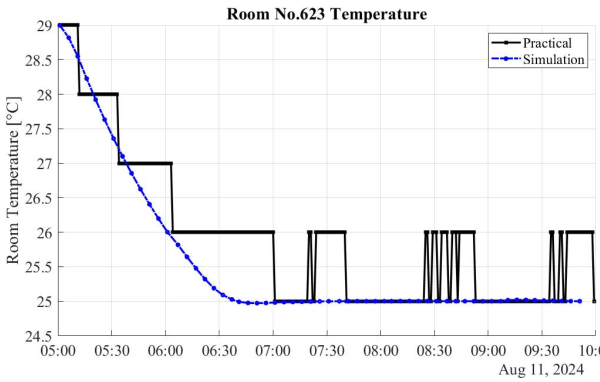

Fig. 21. The temperature regulation performance of the proposed modified version optimization algorithm for the office No. 623 for summer season

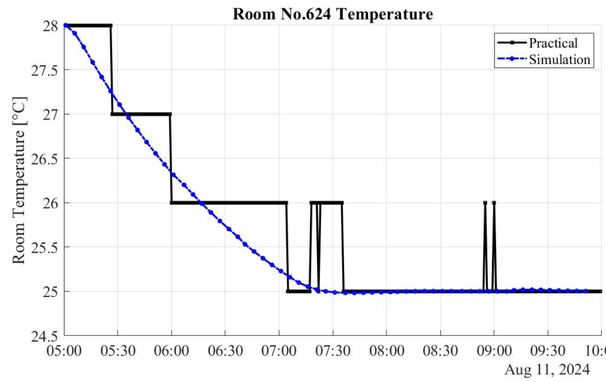

Fig. 23. The temperature regulation performance of the proposed modified version optimization algorithm for the office No. 624 for summer season

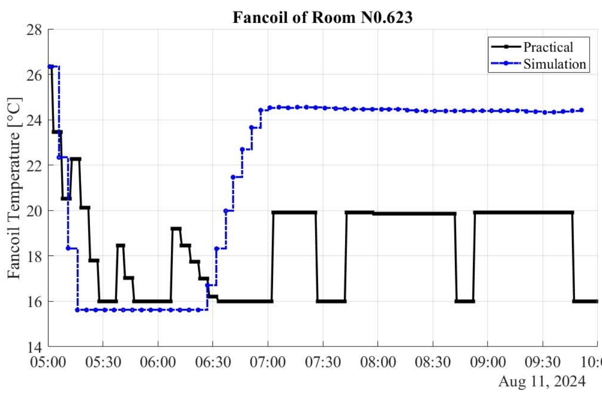

Fig. 22. FCUs temperature set by the modified version optimization algorithm for the office No. 623 for summer season

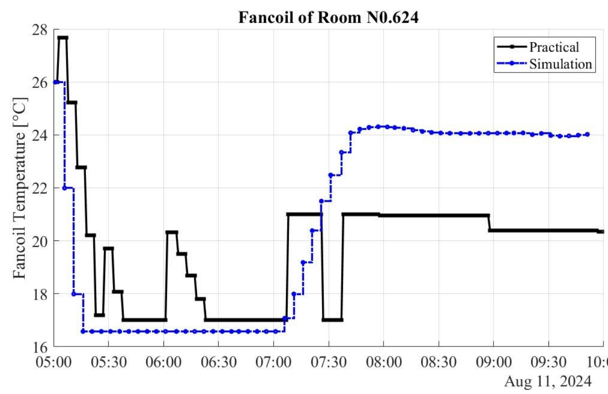

Fig. 24. FCUs temperature set by the modified version optimization algorithm for the office No. 624 for summer season

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